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Reliability-Based Calibration of Design Code for Concrete Structures (ACI 318)

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Abstract

Calibration was performed to determine the resistance factors corresponding to a new set of load and load combination factors. The study involved the development of calibration procedure, development of statistical models for load and resistance, reliability analysis for selected representative structural components and materials, selection of the target reliability indices, and finally selection of the recommended resistance factors. Structural resistance is considered as a product of three factors: material properties, fabrication (dimensions), and professional (analysis factor). The major focus of this study is the analysis of material properties. Test results were provided by the industry for three groups of material: concrete, reinforcing steel bars and prestressing steel strands. It was observed that the quality of materials has been improved over the last 30 years and this is reflected in reduced coefficients of variation and increased bias factors. The reliability is expressed in terms of reliability index calculated as a function of the load and resistance parameters. The results served as a basis for the selection of the target reliability indices. The final step in calibration is the selection of resistance factors.

keyword: Concrete, Resistance factor, Cumulative distribution function, Statistical parameters,

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1 Introduction

In the limit states design, load effect is compared with load carrying capacity (resistance). The limit state function represents a state of equilibrium, when load and resistance balance each other and the difference between load and resistance is a safety margin. Each failure mode can be represented by a limit state function. The load and resistance parameters can involve a considerable degree of uncertainty and should be treated as random variables. Therefore, reliability is a rational measure of structural performance. The design process, known as the Limit States Design, requires a set of load and resistance factors for each limit state. Objective of the code calibration is to select these factors so that the reliability of designed structures is at the predetermined target level.
This paper documents the statistical analysis for concrete, reinforcing steel bars and prestressing steel strands. The statistical parameters showed that the quality of workmanship has improved over the last 20-30 years and this allowed for an increase of resistance parameters.
The calibration process is focused on the reliability analysis procedures. In this study, safety is measured in terms of the reliability index. The developed procedure is applied to calculate the reliability indices for the beams and slabs in shear, for various ratios of load components. The reliability indices corresponding to current practice are presented in the literature (NOWAK and COLLINS, 2000). The target reliability level depends on the consequences of failure, and the cost of increasing/decreasing the safety margin by a unit (marginal cost of safety). The final step is the selection of the resistance factor corresponding to the load factors specified in ACI 318. The resistance factors are rounded to the nearest 0.05. To check the consistency of the results, reliability indices are calculated using the proposed resistance factors and the results are compared with the target values.

2 Load and Resistance Model – General Information

Resistance of a structural component, R, is a function of material properties and dimensions. R is a random variable due to various categories of uncertainties. It is convenient to consider R as a product of three factors,

$$R = R_n \cdot M \cdot F \cdot P \qquad \text{(Equation 1)}$$

where:
R_n- Nominal (design) value of resistance,
M - Materials factor representing material properties, in particular strength and modulus of elasticity,
F - Fabrication factor representing dimensions and geometry of the component, including cross-section area, moment of inertia, and section modulus,

P - Professional factor representing the approximations involved in the structural analysis and idealized stress/strain distribution models. The professional factor P is defined as the ratio of the test capacity to analytically predicted capacity.
The statistical parameters for M, F and P were considered by various researchers and the results were summarized by ELLINGWOOD et al. (1980) based on material test data available in 1970's. However, it has been observed that the quality of materials such as reinforcing steel and concrete has improved over the years. Therefore, in this study, material test database has been updated.
The test data for concrete were obtained from ready mix companies and precasting plants. The lightweight concrete data included 8240 samples, ordinary concrete data about 1110 samples, and high-strength concrete 2050 samples. Whereas, the yield strength of reinforcing bars was tested on total of 15770 samples. For fabrication factor and professional factor, the statistical parameters are taken from the available literature (ELLINGWOOD et al. 1980, REINECK et al. 2003, RAKOCZY and NOWAK 2012). It is assumed that the variability of material properties and dimensions correspond to an average quality of construction expected in practice. Long-term changes in concrete and steel affecting strength are not considered.

The load is considered as a combination of dead load, D, and live load, L (ASCE 7-10). The statistical parameters are assumed based on the available literature (ELLINGWOOD et al 1980; NOWAK and COLLINS 2000). For dead load, the bias factor, λ = 1.05 and coefficient of variation, V = 0.10. For live load, λ = 1.0 and V = 0.18. The reliability analysis is performed for a full range of D and L ratios.

3 Material Factors

The statistical parameters of material factor, M, were determined from the test data provided by the industry in the research conducted by NOWAK et al. 2003, 2010, and 2011. The tests were performed by producers of materials and submitted in the last decade through the associations representing the industry.
The obtained test data were plotted on the normal probability paper. The construction and use of the normal probability paper is described in textbooks (NOWAK and COLLINS 2000). It is a convenient way to present cumulative distribution functions (CDF), as it allows for an easy evaluation of the most important statistical parameters as well as type of the distribution function. The horizontal axis represents the basic variable, i.e. strength of the sample. Vertical axis is the inverse normal probability scale, and it represents the distance from the mean value in terms of standard deviations. The vertical coordinate can also be considered as the probability of exceeding the corresponding value of the variable. For any value of concrete strength (horizontal axis), the vertical coordinate of CDF corresponds to a certain probability of being exceeded. For example, value of 1 on the vertical scale corresponds to 0.159 probability that that value of concrete strength will be exceeded.
The CDF's of f_c' for normal weight concrete are plotted in Fig. 1, for nominal strength of 21 to 45 MPa. For comparison, the CDF's of fc' for high-strength concrete are plotted in Fig.

2, for nominal strength of 48.5 to 82.5 MPa. The CDF's of fc' for lightweight concrete are plotted in Fig. 3, for nominal strength of 21 to 50 MPa.

The industry provided test data of yield strength for the reinforcing steel bars with the nominal yield strength of 420 MPa, and for sizes from #3 to #14 (diameters from 9.5 mm through 44 mm). The CDF's of yield strength are plotted in Fig. 4. It is observed that bias factors and coefficient of variation are practically the same for all rebar sizes, with λ = 1.13 and V = 0.03. For comparison, the bias factor for fy used in previous studies was λ = 1.125, and the coefficient of variation, V = 0.10 (ELLINGWOOD et al. 1980). The difference in coefficient of variation between the new test data and that from1970's can be explained by the fact that all reinforcing steel in the United States is now produced from recycled steel with much more uni-form properties.

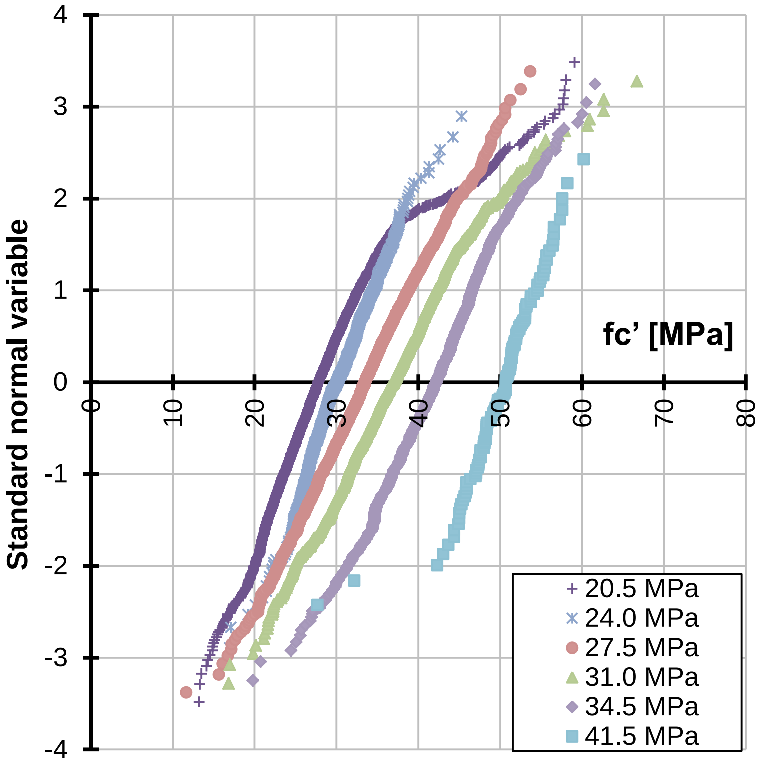

Figure 1 – CDF's of f_c' for Normal Weight Concrete (RAKOCZY and NOWAK 2012)

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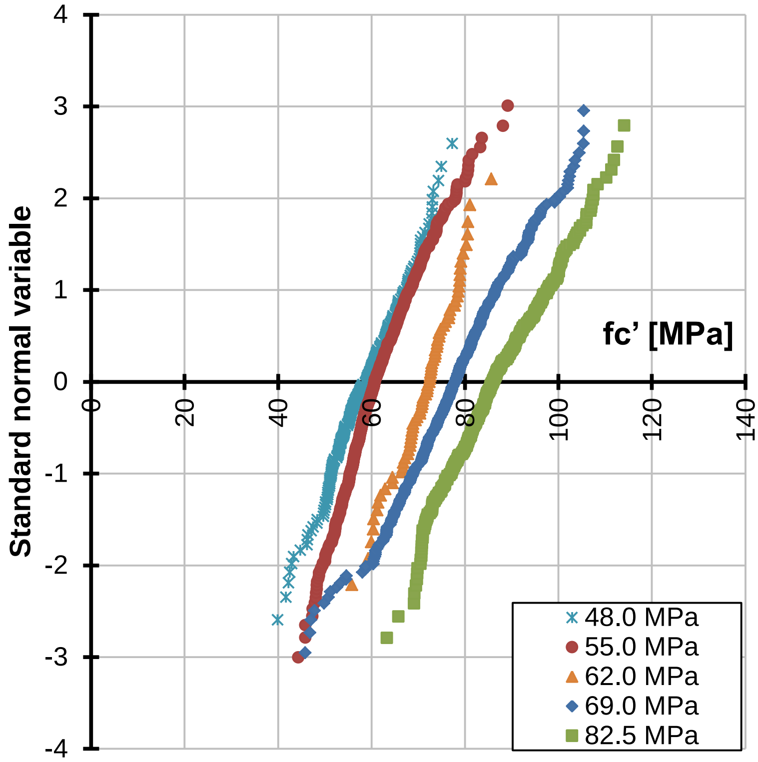

Figure 2 – CDF's of f_c' for High Strength Concrete (RAKOCZY and NOWAK 2012)

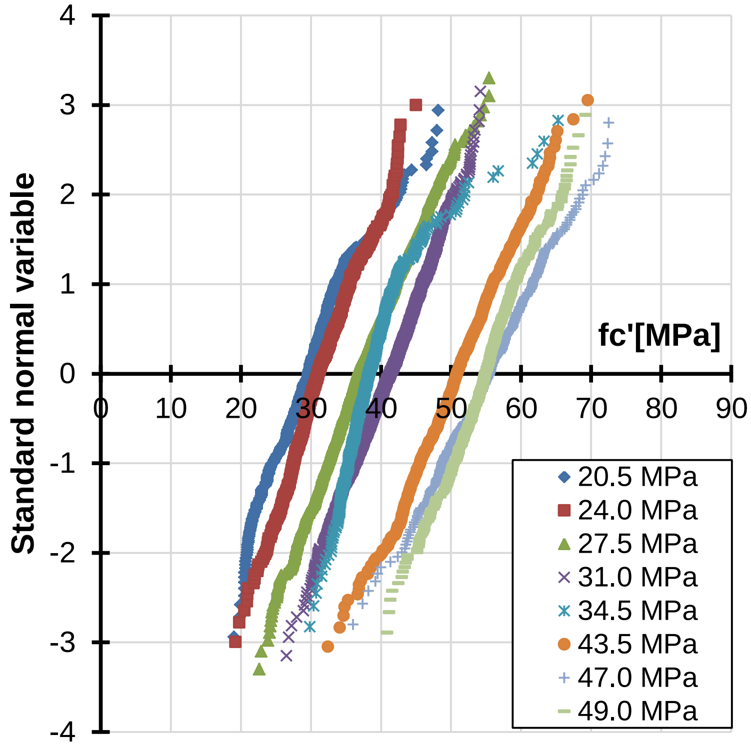

Figure 3 – CDF's of f_c' for Lightweight Concrete (RAKOCZY and NOWAK 2012)

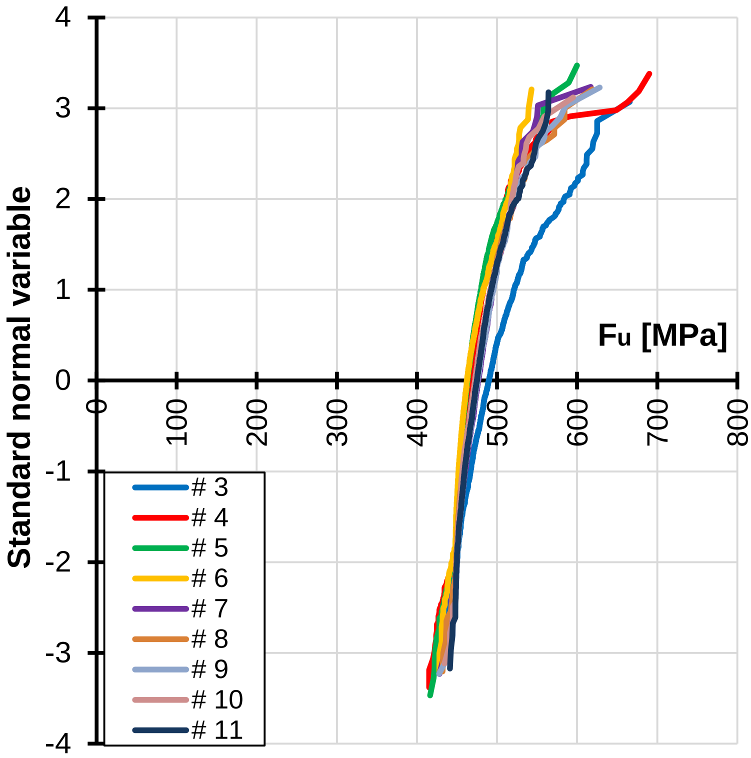

Figure 4 – CDF's of Yield Strength of Rebars (RAKOCZY and NOWAK 2012)

4 Statistical parameters of resistance

The parameters of resistance, R, were taken from the previous study on the material data (RAKOCZY and NOWAK 2012). Material parameters for ordinary and high strength concrete were based on the cylinder test data. The bias factor for concrete strength in structure is reduced compared to the bias factor obtained from cylinder tests, by 10% for moment and 5% for shear. The actual concrete strength in the structure can differ from job to job, but these differences are included in the fabrication and professional factors (λ_F and λ_P). The concrete strength data was obtained from many different sources (concrete mix plants and construction sites) so it includes the batch-to-batch variation, that is higher than within-batch variation. The investigated data also includes variation caused by different testing methods (as data comes from different labs), different mixes and different ingredients.

Material parameters for steel rebars were based on the yield strength data. A formula for resistance (load carrying capacity) is formulated for each of the considered limit states and structural components.

The cumulative distribution function (CDF) of resistance was obtained by generating one million values of R for each considered design case. This served as a basis to calculate the mean of R, m_R, standard deviation, σ_R, bias factor, λ_R, and coefficient of variation, V_R.

The simulations for all selected design cases were performed for normal weight concrete and lightweight concrete, and for various reinforcement ratios. It was found that the reinforcement ratio does not have any significant effect on the parameters of resistance.

The resulting bias factors and coefficients of variation are shown in Table 1 for beams in shear and in Table 2 for slabs in shear of normal weight concrete (NWC) and lightweight concrete (LWC).

Table 1 – Statistical Parameters for R/C Beam – Shear (Rakoczy and Nowak 2012)

Design cases	**NWC – data 2004**		**LWC, data 2009**	
	λ	**V**	**λ**	**V**
Without shear reinforcement; assumption: section carries half of shear				
f_c' = 20.5 MPa (3000 psi)	1.34	0.235	**1.28**	**0.22**
f_c' = 27.5 MPa (4000 psi)	1.27	0.215	**1.23**	**0.21**
f_c' = 34.5 MPa (5000 psi)	1.22	0.20	**1.13**	**0.195**
f_c' = 41.5 MPa (6000 psi)	1.17	0.19	**1.095**	**0.185**
f_c' = 69.0 MPa (10000 psi)	1.115	0.18	**-**	**-**
Minimum code shear reinforcement				
f_c' = 20.5 MPa (3000 psi)	1.30	0.185	**1.22**	**0.175**
f_c' = 27.5 MPa (4000 psi)	1.25	0.17	**1.165**	**0.165**
f_c' = 34.5 MPa (5000 psi)	1.21	0.165	**1.12**	**0.16**
f_c' = 41.5 MPa (6000 psi)	1.18	0.155	**1.095**	**0.155**
f_c' = 69.0 MPa (10000 psi)	1.14	0.15	**-**	
Minimum practical shear reinforcement (2#3 bars)				
f_c' = 20.5 MPa (3000 psi)	1.25-1.27	0.14-0.16	**1.16**	**0.135**
f_c' = 27.5 MPa (4000 psi)	1.23-1.25	0.135-0.155	**1.135**	**0.135**
f_c' = 34.5 MPa (5000 psi)	1.21	0.13-0.15	**1.11**	**0.13**
f_c' = 41.5 MPa (6000 psi)	1.19	0.13-0.145	**1.095**	**0.13**
f_c' = 69.0 MPa (10000 psi)	1.14-1.155	0.13-0.145	**-**	**-**
Shear, average shear reinforcement				
f_c' = 20.5 MPa (3000 psi)	1.225	0.135	**1.15**	**0.13**
f_c' = 27.5 MPa (4000 psi)	1.225	0.13	**1.124**	**0.125**
f_c' = 34.5 MPa (5000 psi)	1.21	0.125	**1.104**	**0.125**
f_c' = 41.5 MPa (6000 psi)	1.195	0.125	**1.095**	**0.125**
f_c' = 69.0 MPa (10000 psi)	1.175	0.12	**-**	**-**

Table 2 – Statistical Parameters for R/C Slab – Shear (Rakoczy and Nowak 2012)

Design cases	NWC – data 2004		LWC, data 2009	
	λ	V	λ	V
One-way shear (slab depth 4, 6, 8 in)				
f_c' = 20.7 MPa (3000 psi)	1.32	0.265	**1.40**	**0.25**
f_c' = 27.6 MPa (4000 psi)	1.26	0.245	**1.31**	**0.235**
f_c' = 34.5 MPa (5000 psi)	1.20	0.23	**1.235**	**0.225**
f_c' = 41.4 MPa (6000 psi)	1.16	0.22	**1.195**	**0.22**
f_c' = 69.0 MPa (10000 psi)	1.11	0.21	**-**	**-**
Two-way shear (slab depth 4, 6, 8 in)				
f_c' = 20.7 MPa (3000 psi)	1.52	0.27	**1.52**	**0.255**
f_c' = 27.6 MPa (4000 psi)	1.44	0.25	**1.42**	**0.24**
f_c' = 34.5 MPa (5000 psi)	1.38	0.24	**1.34**	**0.235**
f_c' = 41.4 MPa (6000 psi)	1.33	0.23	**1.30**	**0.225**
f_c' = 69.0 MPa (10000 psi)	1.26	0.22	**-**	**-**

5 Reliability analysis procedure

Structural performance can be measured in terms of reliability index, β, defined as a function of the probability of failure (NOWAK and COLLINS 2000). The reliability analysis is performed for the limit state functions formulated for the considered structural types and load components. In the calibration process, load and resistance are treated as random variables. The statistical parameters for resistance are based on the data from 2004, and statistical parameters of load are taken from the available literature (ELLINGWOOD et al. 1980).

Formulation of the limit state function requires a definition of failure as the limit state function represents a boundary between desired and undesired performance of a structure. A simple format of the limit state function is:

$$g(R,Q) = R - Q \qquad \text{(Equation 2)}$$

where R represents resistance (capacity) and Q represents the load effect (demand). If g ≥ 0, the structure is safe (structural capacity is greater than load effect); if g < 0, the structure fails (load effect is greater than structural capacity). The probability of failure, P_f, is equal to the probability that the undesired performance will occur and it can be expressed as follows:

$$P_f = P(R - Q < 0) = P(g < 0) \qquad \text{(Equation 3)}$$

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The reliability index for limit state function defined by Equation 3 can be calculate using the following formula:

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \qquad \text{(Equation 4)}$$

where μ_R = mean resistance; μ_Q = mean load effect; σ_R = standard deviation of resistance and σ_Q = standard deviation of load effect.

The analysis was performed for concrete beams and slabs with fc'= 20.5 MPa (3000 psi). The calculated reliability indices are shown in Figure 5 for beam in shear and in Figure 6 for slab in shear.

The design ϕ factor is 0.75 for shear for both beams and slabs. Therefore, for each considered design case, the results are presented for three values of ϕ for lightweight concrete, design value and +/- 0.05. For normal weight concrete only design value of ϕ specified in the ACI 318 Code (2012) was considered.

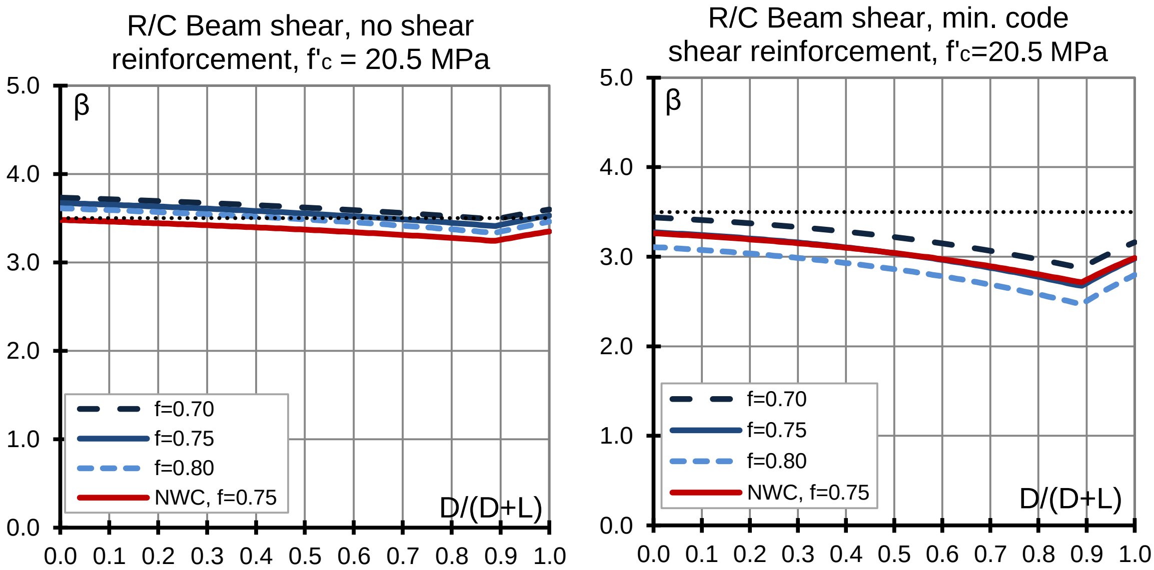

Figure 5 – Reliability Index vs. Load Ratio – R/C Beam, Shear

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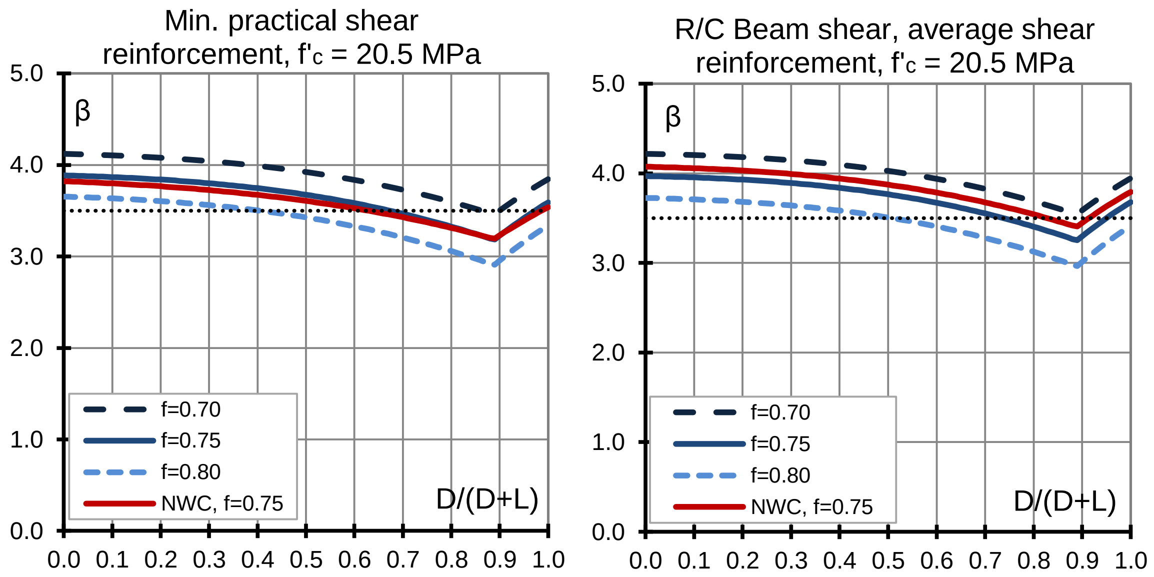

Figure 5 (cont.) – Reliability Index vs. Load Ratio – R/C Beam, Shear

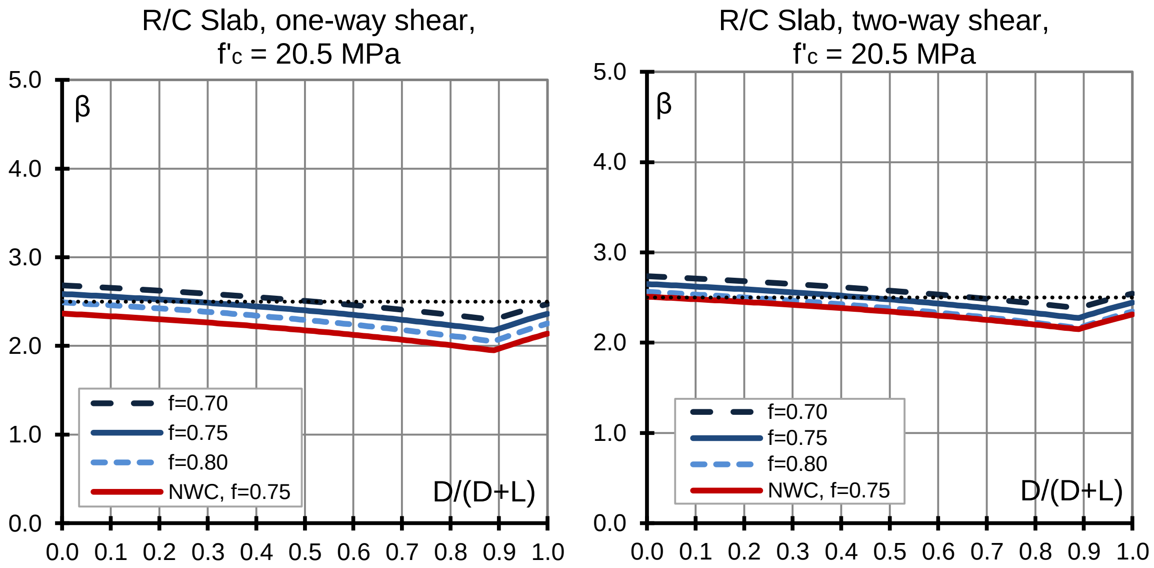

Figure 6 – Reliability Index vs. Load Ratio – R/C Slab, Shear

6 Resistance factor

The reliability analysis was performed for the representative concrete beams for flexure and shear. The reliability indices were calculated for three different values of resistance factor for lightweight concrete and for NWC beams with design resistance factor. It was found that the shear reinforcement has a little effect on the reliability of the considered beams and slabs.

The reliability indices are calculated for the selected representative beams assuming different values of the resistance factor, ϕ, rounded to the nearest 0.05. The reliability indices for lightweight concrete and normal weight concrete are very close to each other. The target beta for beam is 3.5 and for slab is 2.5. It was found that the resistance factor defined in the code provide a sufficient level of safety for both materials. However, it has to be noticed that in the calculations only 20.5 MPa compressive strength of concrete were considered. To confirm this conclusions more cases should be considered.

7 Conclusions

The statistical parameters of material were updated based on new material test data provided by industry. The data include compressive strength of concrete and yield strength of reinforcing steel. Material test data is presented in form of the cumulative distribution functions (CDF) plotted on the normal probability paper for an easier interpretation of the results. It was found that the statistical parameters of compressive strength are slightly better for lightweight concrete compared to that of normal weight concrete. The bias factor for lightweight concrete is slightly higher than that for the ordinary concrete and high strength concrete. Moreover, it was observed that the coefficient of variation of strength of concrete is reduced.This is an indication of a more conservative approach to the application of a relatively new material. The statistical parameters of resistance were taken from the previous study (RAKOCZY and NOWAK 2012).

The reliability indices were calculated for the selected reinforced concrete beams and slabs for several different steel reinforcement ratios and compresive strength of concrete equal 20.5 MPa. The selected criterion for resistance factor was design value and +/- 0.05. The results indicate that the resistance factor defined by the ACI 318 is appropriate in therms of the reliability index.

8 References

ACI 318-08, Building Code Requirements for Structural Concrete, American Concrete Institute, Farmington Hills, Michigan, 2008

ASCE/SEI 7-10, Minimum Design Loads for Buildings and Other Structures, American Society of Civil Engineers, Reston, VA, 2010

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ELLINGWOOD, B., GALAMBOS, T. V., McGregor, J. G., CORNELL, C. A**., Development of a Probability Based Load Criterion for American National Standard A58**, NBS Special Report 577, U.S. Department of Commerce, National Bureau of Standards, 1980.

NOWAK, A. S., COLLINS K. R., **Reliability of Structures**, McGraw-Hill International Editions, Civil Engineering Series, Singapore, 2000

NOWAK, A.S. and RAKOCZY, A.M., **Statistical Parameters for Compressive Strength of Light-Weight Concrete**, Concrete Bridge Conference, PCI, Phoenix, AZ, Feb. 2010, CD

NOWAK, A. S., RAKOCZY, A. M., SZELIGA, E., **Revised Statistical Resistance Models for R/C Structural Components**, ACI SP honoring Andy Scanlon, October 2011

NOWAK, A. S., SZERSZEN, M., **Calibration of Design Code for Buildings (ACI 318): Part 1&2**, ACI Structural Journal, V.100, No.3, May 2003, pp.377-382

RAKOCZY, A.M. and NOWAK, A. S., **Resistance Model of Lightweight Concrete Members**, ACI Structural Journal, 2012 (to appear)

REINECK, K-H.; KUCHMA, D.; KIM, K.S.; and MARX, S., 2003; **Shear Database for Reinforced Concrete Members without Shear Reinforcement**, ACI Structural Journal, Vol. 100, No. 2,